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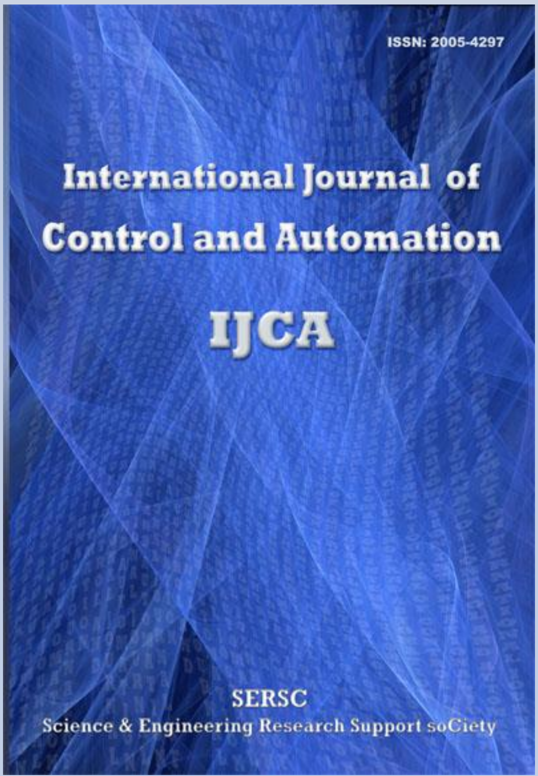

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Differential Transformation for Kinematic Modeling of Autonomous Bicycle

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Abstract

Kinematic models are indispensable for the trajectory planning and the control of autonomous bicycles. The conventional approaches for kinematic models of bicycles mainly depend on the geometrical intuition. The kinematic modeling approach developed in this study is based on the systematic differential motion transformation; thus, it is helpful to understand the bicycle motion at each coordinate system and is applicable to the controller design for various types of bicycles. The differential motion transformation represents the Jacobian relationship between two coordinate frames, which is the velocity kinematics of a bicycle. Computer simulations are conducted to verify the kinematic model and the tracking control for autonomous bicycles in this study.

Keywords*: autonomous bicycle, differential motion transformation, Jacobian, velocity kinematic model, inverse kinematic control*

1. Introduction

The autonomous bicycle has drawn increasing attention as an educational and experimental test bed for the control theory [1-2]. There exists many studies about autonomous bicycles focused on the methods of dynamic modeling and the control [3-4]. For motion planning of the autonomous bicycle, a kinematic model is also indispensable because it provides a basis for a tracking controller design to achieve a given desired trajectory. However, the existing kinematic modeling methods mainly rely on the geometric intuition [1,5], it is difficult to apply these methods to many types of bicycles with various structures [6].

Muir *et al.* developed a kinematic modeling method for general wheeled mobile robots by using a coordinate transformation matrix [7]. For a kind of bicycles, Klein presented the velocity kinematic model based on the geometrical analysis [1]. Similar kinematic models were used to design tracking controllers for autonomous bicycles [8-10]. Ham *et al.* proposed an iterative algorithm for the inverse kinematic problem of autonomous bicycles based on the geometrical kinematic model [5]. Tanaka *et al.* addressed steering control to follow a given trajectory for a bicycle [11]; Meng *et al.* proposed a variable structure bicycle and a kinematic model based on the geometry of the structure [12].

To understand the motion behavior of bicycles, a systematic method of kinematic modeling is required, rather than a heuristic geometrical method. With this motivation, this study aims to develop a new method of kinematic modeling for autonomous bicycles by using the differential motion transformation. Because the theory of the differential motion transformation has been thoroughly developed [13], it is useful for the systematical modeling of the bicycle kinematics and is applicable to many types of bicycles with various structures.

The differential motion represents the translation and rotation with respect to each axis in a coordinate system over a small time interval. Therefore, the transformation of differential motion represents a Jacobian relationship between the coordinate systems.

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This relationship is used in this study to convert the speed of the driving wheel into the velocity of a bicycle: the velocity kinematics of a bicycle. The velocity kinematics can be used inversely to determine the required driving speed of a wheel and the steering angle from the desired trajectory of a bicycle for tracking control. This paper is organized as follows: a brief introduction to the general theory of the differential motion transformation is provided in Section 2. The velocity kinematic model for a bicycle is derived based on the differential motion transformation in Section 3. In Section 4, the kinematic model is verified by computer simulation for tracking control associated with the inverse kinematics and concluding remarks are presented in Section 5.

2. Differential Motion Transformation

The differential motion of a point, including translation and rotation in a coordinate system, A, is represented as follows:

$$\Delta^A = \begin{bmatrix} 0 & -\delta_z^A & \delta_y^A & d_x^A \\ \delta_z^A & 0 & -\delta_x^A & d_y^A \\ -\delta_y^A & \delta_x^A & 0 & d_z^A \\ 0 & 0 & 0 & 0 \end{bmatrix} \tag{1}$$

where the differential motion vectors $\boldsymbol{\delta}^A = \left[\delta_x^A \quad \delta_y^A \quad \delta_z^A\right]^t$ and $\mathbf{d}^A = \left[d_x^A \quad d_y^A \quad d_z^A\right]^t$ are the rotational and translational velocity, respectively, as $\delta t \to 0$ with respect to each axis of the coordinate system, A.

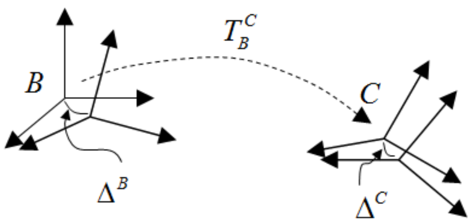


Figure 1. Transformation of Differential Motion between Coordinate Systems

Given two coordinate systems, B and C, and the homogeneous coordinate transformation between them, T_B^C, the differential motion, Δ^B can be transformed into Δ^C shown in Figure 1 as follows:

$$\Delta^C = T_B^{C-1} \Delta^B T_B^C \tag{2}$$

The transformation, T_B^C, can be represented by column vectors, $\mathbf{n}$, $\mathbf{o}$, $\mathbf{a}$, and $\mathbf{p}$ as in (3):

$$\begin{aligned} T_B^C &= \begin{bmatrix} n_x & o_x & a_x & p_x \\ n_y & o_y & a_y & p_y \\ n_z & o_z & a_z & p_z \\ 0 & 0 & 0 & 1 \end{bmatrix} \\ &= \begin{bmatrix} \mathbf{n} & \mathbf{o} & \mathbf{a} & \mathbf{p} \\ 0 & 0 & 0 & 1 \end{bmatrix} \end{aligned} \tag{3}$$

By inserting (3) into (2), the following matrix equation is obtained:

$$\begin{bmatrix} 0 & -\delta_z^C & \delta_y^C & d_x^C \\ \delta_z^C & 0 & -\delta_x^C & d_y^C \\ -\delta_y^C & \delta_x^C & 0 & d_z^C \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & -\boldsymbol{\delta}^B \cdot \mathbf{a} & \boldsymbol{\delta}^B \cdot \mathbf{o} & \mathbf{n} \cdot \left(\boldsymbol{\delta}^B \times \mathbf{p} + \mathbf{d}^B \right) \\ \boldsymbol{\delta}^B \cdot \mathbf{a} & 0 & -\boldsymbol{\delta}^B \cdot \mathbf{n} & \mathbf{o} \cdot \left(\boldsymbol{\delta}^B \times \mathbf{p} + \mathbf{d}^B \right) \\ -\boldsymbol{\delta}^B \cdot \mathbf{o} & \boldsymbol{\delta}^B \cdot \mathbf{n} & 0 & \mathbf{a} \cdot \left(\boldsymbol{\delta}^B \times \mathbf{p} + \mathbf{d}^B \right) \\ 0 & 0 & 0 & 0 \end{bmatrix} \tag{4}$$

In (4), "·" and "×" represent inner and outer products of two vectors, respectively. The corresponding terms between the left and right sides of (4) are arranged as follows:

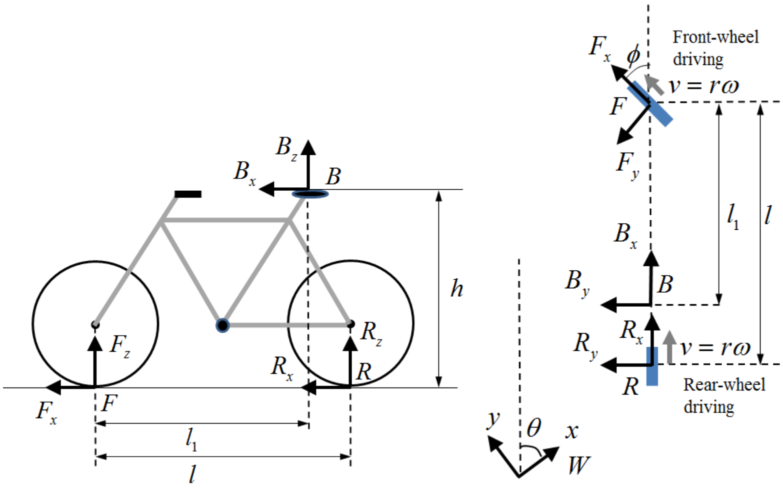
$$\begin{aligned} \delta_x^C &= \boldsymbol{\delta}^B \cdot \mathbf{n} \\ \delta_y^C &= \boldsymbol{\delta}^B \cdot \mathbf{o} \\ \delta_z^C &= \boldsymbol{\delta}^B \cdot \mathbf{a} \\ d_x^C &= \mathbf{n} \cdot \left(\boldsymbol{\delta}^B \times \mathbf{p} + \mathbf{d}^B \right) \\ d_y^C &= \mathbf{o} \cdot \left(\boldsymbol{\delta}^B \times \mathbf{p} + \mathbf{d}^B \right) \\ d_z^C &= \mathbf{a} \cdot \left(\boldsymbol{\delta}^B \times \mathbf{p} + \mathbf{d}^B \right) \end{aligned} \tag{5}$$

These represent the transformation of differential motion between coordinate systems [13].

3. Kinematic Model of Bicycle by Using Differential Motion Transformation

According to the driving wheel, there are two types of bicycles: those driven by the front wheel and those driven by the rear wheel [9, 18]. Because the velocity vector has different interpretation at each coordinate frame, those bicycles have different velocity kinematics. In this section, the velocity kinematics is obtained for each type of bicycle by using the differential motion transformation in this section. The velocity kinematics implies the relationship between the angular speed of the driving wheel and the velocity of the bicycle's reference position with respect to the world coordinates fixed on the ground.

The structure of a standard bicycle and assignment of the coordinate system are shown in Figure 2. In the figure, F and R represent the coordinate systems of the front and rear wheels at contact points with the ground. The world coordinate system is W and the moving coordinate system, B, is set at the reference position of the bicycle, *e.g.*, the rider's position on the bicycle. The radius of each wheel and the angular speed of the driving wheel are denoted as r and ω. In Figure 2 (b), the steering angle between F_x and R_x and the heading angle between R_x and W_x are represented as ϕ and θ, respectively.



(a) Bicycle (Side View) (b) Assignment of Coordinates (Top View)

Figure 2. Coordinate System of Autonomous Bicycle

3.1. Rear-wheel Driving

In the case of rear-wheel driving, the actuation, ω, of the rear wheel leads to the driving speed, $v_x^R = r\omega$, along the R_x axis. The lateral speed of the rear wheel is $v_y^R = 0$. The transformation of differential motion between R and F is described as follows from (2):

$$\Delta^F = T_R^{F-1} \Delta^R T_R^F \tag{6}$$

where Δ^F and Δ^R denote the differential motions in F and R. The transformation, T_R^F, is represented as

$$T_R^F = Trans(x,l)\,Rot(z,\phi) = \begin{bmatrix} c\phi & -s\phi & 0 & l \\ s\phi & c\phi & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{7}$$

In (7), $Rot(\cdot,\cdot)$ and $Trans(\cdot,\cdot)$ denote the rotational and translational transformations, respectively; $s\phi$ and $c\phi$ imply $\sin\phi$ and $\cos\phi$.

According to (3) and (5) combined with (7), the differential motion in F is described as follows:

$$\delta_x^F = c\phi\,\delta_x^R + s\phi\,\delta_y^R \tag{8-1}$$

$$\delta_y^F = -s\phi\,\delta_x^R + c\phi\,\delta_y^R \tag{8-2}$$

$$\delta_z^F = \delta_z^R \tag{8-3}$$

$$d_x^F = c\phi\, d_x^R + l\, s\phi\,\delta_z^R + s\phi\, d_y^R \tag{8-4}$$

$$d_y^F = -s\phi\, d_x^R + l\, c\phi\,\delta_z^R + c\phi\, d_y^R \tag{8-5}$$

$$d_z^F = d_z^R - l\,\delta_y^R \tag{8-6}$$

Because the motion of the front wheel is composed of the rotational component about the z-axis and the translational component on the $x-y$ plane, only the equations (8-3), (8-4) and (8-5) about δ_z^F, d_x^F, and d_y^F are considered. The lateral motion of the rear wheel in R is $d_y^R = 0$ and that of the front wheel in F is also $d_y^F = 0$. Thus, (8-5) becomes

$$\delta_z^R = \frac{1}{l}\tan\phi\, d_x^R \tag{9}$$

Inserting (9) into (8-4) and (8-5) gives the following differential motion in F:

$$\delta_z^F = \frac{1}{l}\tan\phi\, d_x^R \tag{10-1}$$

$$d_x^F = \frac{1}{c\phi} d_x^R \tag{10-2}$$

The differential motion in (10) together with $d_y^F = 0$ corresponds to the following velocity as $\delta t \to 0$:

$$\omega_z^F = \frac{1}{l}\tan\phi\, v_x^R \tag{11-1}$$

$$v_x^F = \frac{1}{c\phi} v_x^R \tag{11-2}$$

$$v_y^F = 0 \tag{11-3}$$

It is possible to transform the velocity in F into those in W by using rotational transformation about the z-axis as follows:

$$\begin{aligned}\left.\begin{bmatrix} v_x^F \\ v_y^F \\ \omega_z^F \end{bmatrix}\right|_W &= \begin{bmatrix} c(\theta+\phi) & -s(\theta+\phi) & 0 \\ s(\theta+\phi) & c(\theta+\phi) & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \dfrac{v_x^R}{c\phi} \\ 0 \\ \dfrac{\tan\phi}{l} v_x^R \end{bmatrix} \\ &= r\omega \begin{bmatrix} \dfrac{c(\theta+\phi)}{c\phi} \\ \dfrac{s(\theta+\phi)}{c\phi} \\ \dfrac{\tan\phi}{l} \end{bmatrix}\end{aligned} \tag{12}$$

where $\theta+\phi$ is the angle between W_x and F_x, as shown in Figure 2 (b). Eq. (12) is the representation of the front wheel velocity in W.

It is possible to obtain the velocity of the reference position, B, of the bicycle from Δ^F as follows: the differential motion, Δ^F, is transformed into Δ^B by using (13):

$$\Delta^B = T_F^{B-1} \Delta^F\, T_F^B \tag{13}$$

where T_F^B is given as

$$\begin{aligned} T_F^B &= Rot(z,-\phi)\ Trans(x,-l_1)\ Trans(z,h) \\ &= \begin{bmatrix} c\phi & s\phi & 0 & -l_1 c\phi \\ -s\phi & c\phi & 0 & l_1 s\phi \\ 0 & 0 & 1 & h \\ 0 & 0 & 0 & 1 \end{bmatrix} \end{aligned} \tag{14}$$

As before, by using (3), (5), and (14), the translational velocity on the $x-y$ plane and the rotational velocity about the z-axis in B are obtained as follows:

$$v_x^B = v_x^R \tag{15-1}$$

$$v_y^B = \left(1 - \frac{l_1}{l}\right) \tan\phi \, v_x^R \tag{15-2}$$

$$\omega_z^B = \frac{1}{l} \tan\phi \, v_x^R \tag{15-3}$$

Representation of (15) in W is obtained as (16):

$$\left. \begin{bmatrix} v_x^B \\ v_y^B \\ \omega_z^B \end{bmatrix} \right|_W = \begin{bmatrix} c\theta & -s\theta & 0 \\ s\theta & c\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} v_x^R \\ \left(1 - \frac{l_1}{l}\right) \tan\phi \, v_x^R \\ \frac{\tan\phi}{l} v_x^R \end{bmatrix}$$
$$= r\omega \begin{bmatrix} c\theta - \left(1 - \frac{l_1}{l}\right) s\theta \tan\phi \\ s\theta + \left(1 - \frac{l_1}{l}\right) c\theta \tan\phi \\ \frac{\tan\phi}{l} \end{bmatrix} \tag{16}$$

3.2. Front-wheel Driving

In the case of front-wheel driving, the driving speed of the front wheel is $v_x^F = r\omega$ in F, as shown in Figure 2 (b). The lateral velocity of the front wheel is $v_y^F = 0$. The differential motion in F can be transformed into R as

$$\Delta^R = T_F^{R-1} \Delta^F \, T_F^R \tag{17}$$

where T_F^R is described as

$$T_F^R = Rot(z, -\phi) \; Trans(x, -l)$$
$$= \begin{bmatrix} c\phi & s\phi & 0 & -lc\phi \\ -s\phi & c\phi & 0 & ls\phi \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{18}$$

By using (5), the translational differential motion on the $x-y$ plane and the rotational differential motion about the z -axis in R are obtained as follows:

$$\delta_z^R = \delta_z^F \tag{19-1}$$

$$d_x^R = c\phi \, d_x^F - s\phi \, d_y^F \tag{19-2}$$

$$d_y^R = s\phi \, d_x^F + c\phi \, d_y^F - l \, \delta_z^F \tag{19-3}$$

Because the lateral motions of two wheels are $d_y^R = 0$ and $d_y^F = 0$, (19-3) becomes

$$\delta_z^F = \frac{1}{l} s\phi \, d_x^F \tag{20}$$

By inserting (20) into (19-1) and (19-2) and representing the resultant equations as $\delta t \to 0$, the following velocity kinematics in R are obtained as

$$\omega_z^R = \frac{1}{l} s\phi \, v_x^F \tag{21-1}$$

$$v_x^R = c\phi \, v_x^F \tag{21-2}$$

$$v_y^R = 0 \tag{21-3}$$

To represent (21) into W, $Rot(z, \theta)$ is premultiplied as follows:

$$\left.\begin{bmatrix} v_x^R \\ v_y^R \\ \omega_z^R \end{bmatrix}\right|_W = \begin{bmatrix} c\theta & -s\theta & 0 \\ s\theta & c\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} c\phi v_x^F \\ 0 \\ \dfrac{s\phi}{l} v_x^F \end{bmatrix} = r\omega \begin{bmatrix} c\theta c\phi \\ s\theta c\phi \\ \dfrac{s\phi}{l} \end{bmatrix} \tag{22}$$

The differential motion in the reference position, B, of the bicycle can be obtained by the following transformation:

$$\Delta^B = T_R^{B-1} \Delta^R T_R^B \tag{23}$$

where T_R^B is given as

$$T_R^B = Trans(x, l - l_1)\ Trans(z, h) = \begin{bmatrix} 1 & 0 & 0 & l - l_1 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & h \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{24}$$

As before, the translational velocity on the $x - y$ plane and the rotational velocity about the z-axis in B as $\delta t \to 0$ are obtained by using (3), (5), and (24) as follows:

$$\omega_z^B = \frac{1}{l} s\phi v_x^F \tag{25-1}$$

$$v_x^B = c\phi v_x^F \tag{25-2}$$

$$v_y^B = \left(1 - \frac{l_1}{l}\right) s\phi v_x^F \tag{25-3}$$

The equations of motion, (25), is represented in W as follows:

$$\left.\begin{bmatrix} v_x^B \\ v_y^B \\ \omega_z^B \end{bmatrix}\right|_W = \begin{bmatrix} c\theta & -s\theta & 0 \\ s\theta & c\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} c\phi v_x^F \\ \left(1 - \dfrac{l_1}{l}\right) s\phi v_x^F \\ \dfrac{s\phi}{l} v_x^F \end{bmatrix} = r\omega \begin{bmatrix} c(\theta + \phi) + \dfrac{l_1}{l} s\theta s\phi \\ s(\theta + \phi) - \dfrac{l_1}{l} c\theta s\phi \\ \dfrac{s\phi}{l} \end{bmatrix} \tag{26}$$

To summarize from (16) and (26), the motion equations of the rider's coordinate system, B, are described in W as follows:

In the case of rear-wheel driving at speed $v = r\omega$:

$$\begin{bmatrix} v_x^B \\ v_y^B \\ \omega_z^B \end{bmatrix}\Bigg|_W = r\omega \begin{bmatrix} c\theta - \left(1-\frac{l_1}{l}\right)s\theta\tan\phi \\ s\theta + \left(1-\frac{l_1}{l}\right)c\theta\tan\phi \\ \frac{\tan\phi}{l} \end{bmatrix} \tag{27}$$

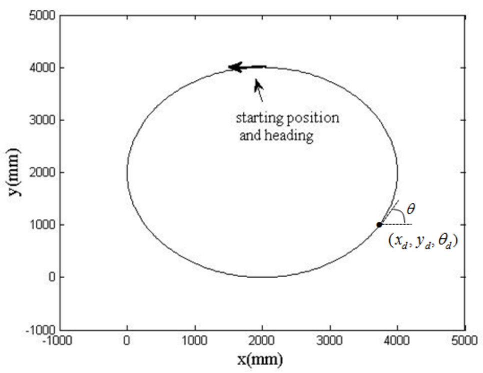
In the case of front-wheel driving at speed $v = r\omega$:

$$\begin{bmatrix} v_x^B \\ v_y^B \\ \omega_z^B \end{bmatrix}\Bigg|_W = r\omega \begin{bmatrix} c(\theta+\phi) + \frac{l_1}{l}s\theta s\phi \\ s(\theta+\phi) - \frac{l_1}{l}c\theta s\phi \\ \frac{s\phi}{l} \end{bmatrix} \tag{28}$$

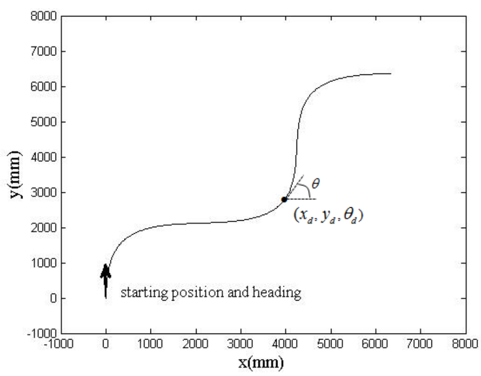
In the preceding equations, the kinematic motion equation of the rear-wheel coordinate system is obtained by inserting l into l_1, which matches the results established by Klein and Ham [1, 8]. It is asserted here that the kinematic models in (27) and (28) are more informative because they explain the velocity vector experienced at each coordinate frame.

4. Computer Simulation

In this section, computer simulations are conducted to verify the proposed kinematic model of a bicycle. An inverse kinematics is considered as a tracking controller to determine the speed of the driving wheel and the steering angle to follow a given desired trajectory. For brevity, only the case of driving by the rear wheel is considered in the simulation. The lengths of the bicycle are set as $l = 1{,}500$ mm and $l_1 = 1{,}000$ mm. The desired trajectories of the circular and curved shapes in the world coordinate system for the rear wheel, $(x_d^R, y_d^R, \theta_d^R)|_W$, are shown in Figure 3. The desired trajectories contain velocity information of $(v_{xd}^R, v_{yd}^R, \omega_{zd}^R)|_W$ as $v_{xd}^R(k)|_W \approx \frac{x_d^R(k)|_W - x_d^R(k-1)|_W}{\Delta t}$, $v_{yd}^R(k)|_W \approx \frac{y_d^R(k)|_W - y_d^R(k-1)|_W}{\Delta t}$, and $\omega_{zd}^R(k)|_W \approx \frac{\theta_d^R(k)|_W - \theta_d^R(k-1)|_W}{\Delta t}$ with small Δt value. For each variable, k is the time index and the subscript d implies the desired value.



(a) Circular Trajectory



(b) Curved Trajectory

Figure 3. Desired Trajectories for Rear Wheel

The velocity kinematics of the rear wheel can easily be obtained by substituting l for l_1; *i.e.*, $l_1 = l$ in (27), which gives

$$\begin{bmatrix} v_x^R \\ v_y^R \\ \omega_z^R \end{bmatrix}\Bigg|_W = r\omega \begin{bmatrix} c\theta \\ s\theta \\ \dfrac{\tan\phi}{l} \end{bmatrix} \tag{29}$$

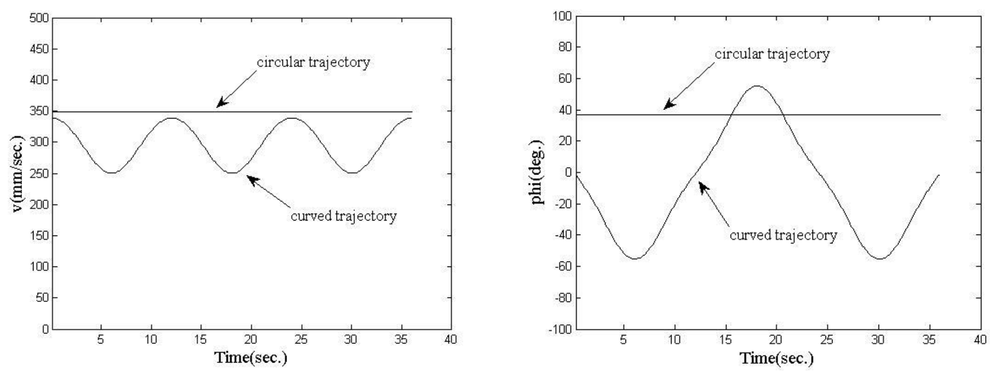
From (29), the driving speed, $v_x^R = r\omega$, of the rear wheel and the steering angle, ϕ, should be as follows to achieve the desired trajectory:

$$v_x^R = \sqrt{\left(v_{xd}^R\big|_W\right)^2 + \left(v_{yd}^R\big|_W\right)^2} = r\omega \tag{30-1}$$

$$\phi = \tan^{-1}\left(\frac{l\,\omega_{zd}^R\big|_W}{v_x^R}\right) \tag{30-2}$$

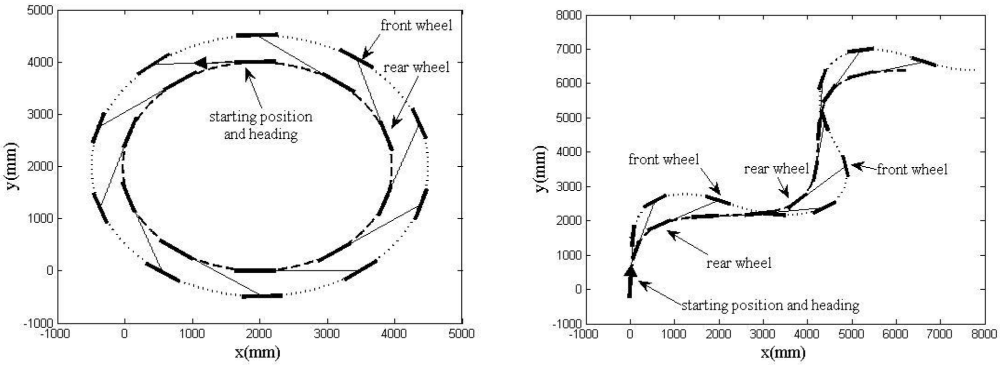
Figure 4 shows the driving speed and the steering angle obtained by (30-1) and (30-2), respectively. For a circular trajectory, the driving velocity and steering angle are constant with time, which is easily understandable.

The resultant trajectories of the front and rear wheels by the driving speed (30-1) and the steering angle (30-2) are given from (12) and (29) as shown in Figure 5. The trajectory of the rider's position is almost indistinguishable from that of the rear wheel and is not presented in Figure 5.



(a) Driving Speed $v_x^R = r\omega$ (b) Steering Angle ϕ

Figure 4. Driving Speed and Steering Angle



(a) Circular Trajectory (b) Curved Trajectory

Figure 5. Resultant Trajectories of Front and Rear Wheels

5. Conclusion

Kinematic modeling is essential for trajectory planning and the control of autonomous bicycles. The kinematic model helps to determine the driving speed of a wheel and the steering angle to follow a given desired trajectory. The conventional kinematic modeling method for bicycles depends on geometrical intuition, and it is difficult to apply various types of bicycles. In this study, a systematic method of kinematic modeling for bicycles is proposed based on the transformation of differential motion. Because the velocity vector has different interpretation at each coordinate frame, the differential motion transformation is useful to obtain the motion vector at each position of a bicycle; the front,–the rear wheels, and the rider’s position.

To verify the velocity kinematic model, computer simulation was conducted for the inverse kinematic tracking control. The proposed method can be used as a unified approach to the kinematic modeling for various types of bicycles.

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