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Convolution-based Time Optimal Path Planning for a High Curvature Bezier Curve Considering Possible Physical Limits

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Abstract

This paper proposes a time optimal path planning while considering the physical constraints of wheeled mobile robots along a high curvature Bezier curve. The proposed algorithm overcomes drawbacks of preceding results. A trajectory produced through acceleration limits showed long traveling time, non-uniform sampling time and unsatisfied terminal velocity. A convolution-based method to consider physical limits was able to meet the velocity requirements, but it demonstrated inability to follow a high curvature path. Therefore, a method based on convolution operator is presented to generate periodic velocity commands and achieve time optimality while satisfying physical limits for a high curvature path.

Keywords: *High Curvature Path, Time Optimal, Velocity Limits, Convolution, Bezier Curve*

1. Introduction

The development of mobile robot systems made it easier for the physical unit to move and react to different situations from simple user-generated velocity commands. This opened up a wide variety of usage such as cleaning robots and intelligent service robots [1].

Supposing that a wheeled robot will be moved from an initial point to an exact target position, a predetermined path should be acknowledged for the robot to follow. Different approaches were made for the path planning that involves different types of curves such as cubic splines [2], cubic spirals [3], and Bezier curves [1, 4-5]. Another crucial factor to be considered during path planning is satisfying the physical constraints of a mobile robot. Any violation to these limitations would make the robot vulnerable to potential damage and produce inaccurate trajectory. Thus, various researches were made in order to deal with these limits such as the maximum velocity, maximum acceleration, and maximum jerk [1, 3-4, 6].

In addition, the calculated curvature from a given path is another limit that should not be neglected. In a curve contemplating high curvature, controlling the robot would be difficult and could result to either; the robot overshooting and not reaching the target point, or for it to slow down—less than the desired terminal velocities. Smooth path planning has been tackled actively between mobile robot researchers [3, 5, 7-8]. Nonetheless, the ability of mobile robots to follow a certain path regardless of its curvature could open new doors for development, especially in the field of obstacle avoidance. In many service applications, mobile robots are required to share their work regions with different obstacles [9].

A path planning method using Bezier curves considering acceleration limits was suggested in [4]. Although this approach could make a robot follow a predetermined path

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with high curvature, huge difference between the actual and required terminal velocities occurred and was shown through simulation results. Moreover, the generated velocity profile compromises only the acceleration constraints of a robot without considering either the maximum velocity or maximum jerk limit. Also, the sampling time of the generated trajectory was non-periodic, which is difficult to be realized in practical applications; cyclic tasks in real-time systems, for example.

This paper utilizes a cubic Bezier curve in path planning and proposes a method to satisfy the physical constraints of the mobile robot for a high curvature path. Even though the central velocity of the robot generated using a convolution operator [10] is able to compromise the physical limits of the mobile robot, it did not consider the direction angle for a Bezier path because the convolution-based method only considers the linear distance between two points. In [1], a path planning algorithm was suggested to consider the direction angle. The modified velocity shows high accuracy in meeting the required terminal velocity however, the trajectory could not follow a planned path with high curvature.

Therefore, we have to consider the curvature when applying the convolution operator meaning that adjustment of the sampling time is required. In order to make a trajectory with uniform sampling time, we apply an approximation method for the modified convolution-based robot trajectory following a high curvature Bezier path in equal sampling time. After calculating the non-uniform time differential of the modified convolution velocity, a new velocity profile is to be generated by linear interpolating the modified velocity values with respect to a uniformly created sampling time. Eventually, the newly formulated trajectory would be defined characteristically as to be driven with periodic velocity commands while staying within the physical limits of the mobile robot.

The effectiveness of the proposed method was determined through various numerical simulations. Trajectories using three different approaches—using acceleration limits [4], modified convolution method [10], and the proposed method—were generated and the results were compared in the sense of time optimality, physical limits and sampling time. According to the results, the previous methods have shown difficulties in following predetermined values in high curvature as expected. On the other hand, the suggested approach has travelled the designed path in the shortest time, while demonstrating periodicity and being within the physical constraints of the mobile robot at the same time. The proposed trajectory generation method could be used as the backbone for future studies in path planning for real time obstacle avoidance.

2. Path Planning Based on a Cubic Bezier Curve

Kinematics is the fundamental of how mechanical systems behave that plays a greater role in following a desired trajectory [11]. In a Cartesian coordinate system, the mobile robot is represented by its central position and its direction angle which is classified in both world frame and robot frame coordinate systems and is defined by the following:

$$P_c = \begin{bmatrix} x_c & y_c & \theta_c \end{bmatrix}^T \tag{1}$$

where x_c and y_c define the position of the robot in the x and y axes, respectively. The heading angle is represented by θ_c.

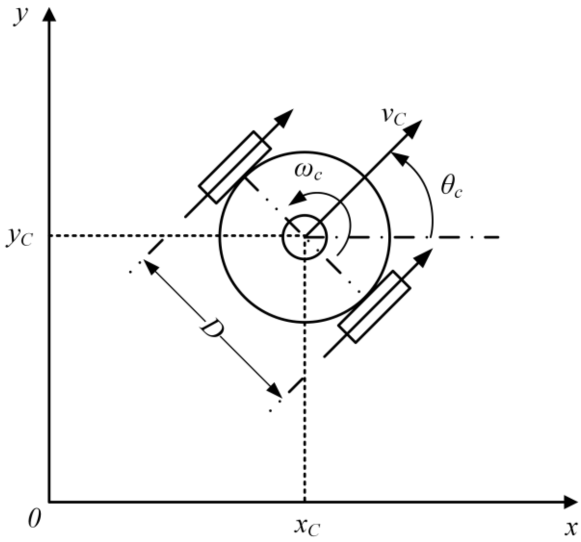

Figure 1. Kinematics of a Mobile Robot

Bezier curve is a parametric curve named after the French engineer, Pierre Bezier. The Bezier curve differs from other type of curves since it considers the heading angle of both its end points and does not have to pass through all the control points that define it, instead, the curve is formed with respect to the polygon that the vertices are described by the data points [4]. The number of the control points are determined by n+1, where n is the degree of the Bezier function. In path planning of a mobile robot, the initial and terminal position with their corresponding heading angles should all be considered. A cubic Bezier curve is generated from an initial point $P(A_0, B_0)$, end point $S(A_3, B_3)$, and two control points that represented by $Q(A_1, B_1)$ and $R(A_2, B_2)$ as found in [1].

3. Velocity Profile Considering Physical Limits

3.1 Time Optimal Path Planning Considering Acceleration Limits

Time optimal path planning requires robots to drive with high speed [2]. Thus, a smooth path is necessary in order to satisfy this requirement. The smoothness of a curve is determined by its curvature. The curvature also means the inverse of the radius of curvature; the radius of the circle that defines the curvature, ρ. To generate the velocity profile along the Bezier curve, the velocity is limited by the physical acceleration constraints of the robot, both radial and tangential denoted as a_r and a_t respectively in the following equations:

$$a_r = \frac{v(u)^2}{\rho} \tag{2-a}$$

$$a_t = \frac{dv(u)}{dt} \tag{2-b}$$

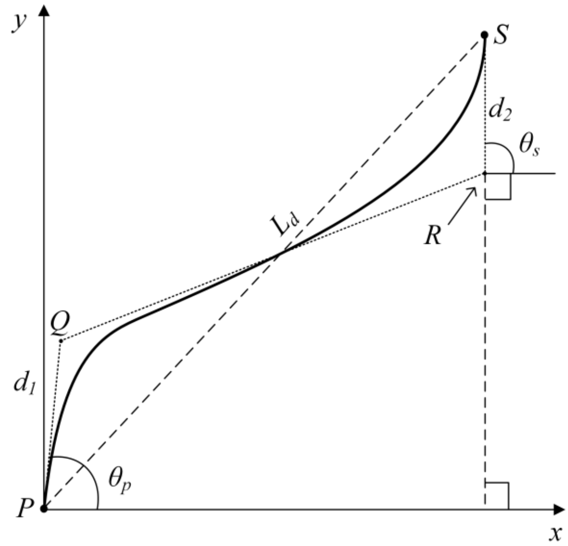

Figure 2. Path Based on a Cubic Bezier Curve

The velocity profile of the mobile robot is determined by the curvature of the generated curve. If the radial acceleration in a certain point within the path is less than the physical limit, then it is said to be realizable in practical situation. In order to satisfy the radial acceleration limits, the calculations are started from either the initial, final, or turning points (local extrema of the curvature) [2]. The maximum possible tangential acceleration is computed considering the radial acceleration values on these points and by using (3):

$$1 = \frac{a_t^{\,2}}{a_{t_{\max}}^{\,2}} + \frac{a_r^{\,2}}{a_{r_{\max}}^{\,2}} \tag{3}$$

Velocity at the next sampling point can be evaluated assuming that the robot is accelerated uniformly between each points and this procedure is continued until the end point. It is suggested to calculate the velocity profile from the starting point in the forward direction, whilst that from the end point is in the backward direction. Moreover, after evaluating the maximum allowable velocity in each turning points using (2-a), the first portion of the velocity profile in each turning points are calculated in the backward direction from the turning point to the initial point, then forward direction to the end point. The maximum allowable velocity profile without violating the acceleration limits is defined by the minimum velocity at each point of the curve from the combination of the calculated velocity profiles [4].

The time duration of the mobile robot to accomplish the generated path is calculated by:

$$t = \int_0^1 \frac{ds(u)}{v(u)} \tag{4}$$

where:

$$ds(u) = \sqrt{\left(\frac{dx(u)}{du}\right)^2 + \left(\frac{dy(u)}{du}\right)^2}\, du \tag{5}$$

3.2. Modified Convolution-based Velocity Profile Generation

A research has shown that a convolution operator is to be used in path planning in order to generate the central velocity trajectory of a mobile robot without violating any physical limits [10]. In order to create the velocity profile using convolution, physical limits should be identified such as the maximum velocity, maximum acceleration, and maximum jerk. The level of the position derivative limit to be satisfied depends on the degree of the curve used in path planning. Figure 3 shows that the velocity profile is generated from a differentiable S-curve such as the cubic Bezier curve, while considering the maximum velocity limit v_{max} in order for the robot to be actuated in a calculated distance, S.

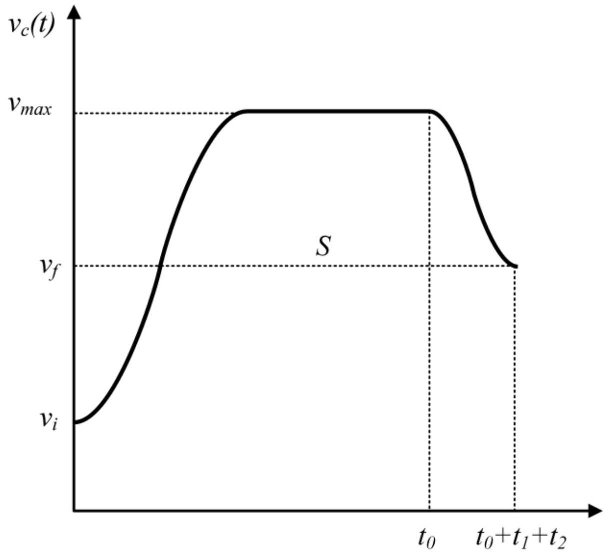

Figure 3. Central Velocity Based on Convolution

Furthermore, time duration of each convolution functions is determined as follows [10]:

$$t_k = \frac{v_{\max}^{(k-1)}}{v_{\max}^{(k)}} \quad for\ k = 0,1,2,..n \tag{6}$$

where the index k depends on the degree of the curve used in path planning.

The central velocity generated using the convolution operator does not consider the angular disposition of the mobile robot [1]. Thus, in order to determine its position while travelling along a Bezier curve, the parameter $u(t)$ is calculated using (7):

$$u(t) = \sum_{t=0}^{t_0+t_1+t_2} \frac{v_{conv}(t)}{B_d} \tag{7}$$

where $u(t)$ is defined as $0 \leq u(t) \leq 1$ and $v_{conv}(t)$ is the original central velocity generated using convolution, and B_d denotes the actual distance travelled along the path designed with Bezier curve.

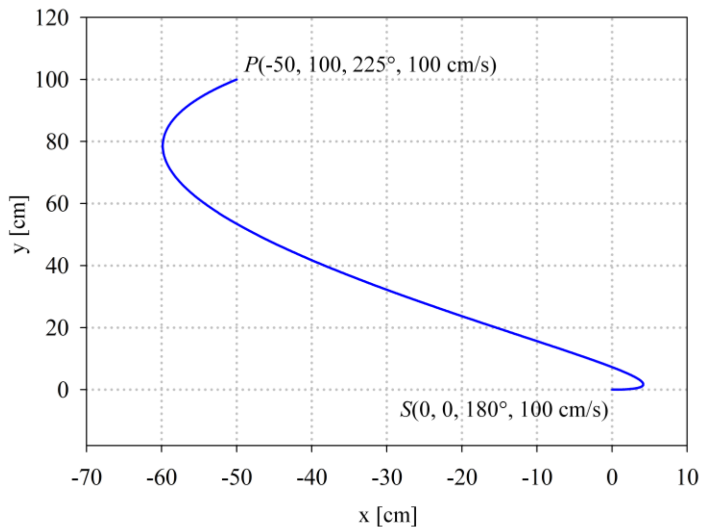

Figure 4. Planned Path Based on Bezier Curve

4. Proposed Method and Simulation Results

A high curvature cubic Bezier curve defined with four control points was used as the path for generating the trajectories using three different methods. According to the results of K.G. Jolly *et al*. [4], the first and third sides of the polygon shown in Figure 2 that surrounds the curve are optimized as d_1=62.84 *cm* and d_2=27.79 *cm*.

As shown in Figure 4, the initial and terminal position and orientation of the robot is located at *P*(-50, 100, 225°) and *S*(0, 0, 180°) respectively. The required velocities on both end points were set to the same value of 100 *cm/s*. The total travel length of the curve was calculated to be 131.9 *cm* by the integration of (5).

Figure 5 shows the velocity profile generated using the first method—considering acceleration limits of the mobile robot. The maximum radial and tangential acceleration of the robot was configured to be 400 cm/s^2 and 200 cm/s^2, same values calculated by the researches of previous studies in [2,4].

Generation of these velocity profiles was discussed in Section 3.1. The minimum velocity for each sampling point corresponded to be the maximum allowable velocity for the robot to travel along the planned path. The flat portion in the figure is said to have values where the acceleration limits were violated. A huge difference between the actual and required terminal velocities is evident in this simulation result. This disparity is due to the high curvature in Turning Point 2, which is less than the expected terminal velocity.

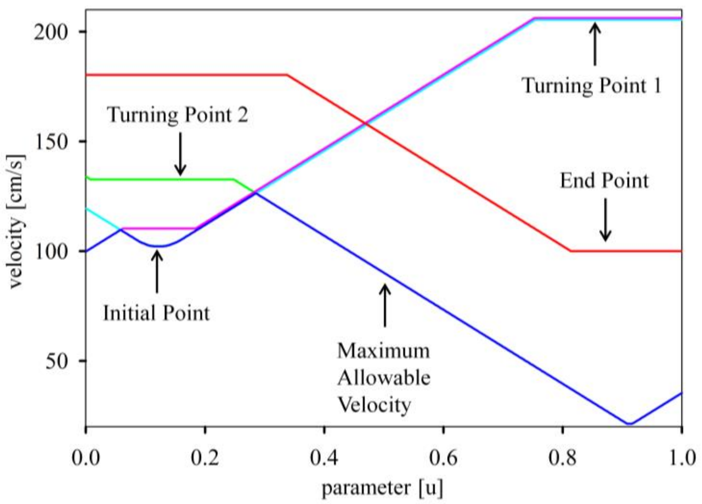

Figure 5. Velocity Profiles Generated from the First Method

The travel time computed was 1.74 s without curvature adjustments and the maximum velocity of the maximum allowable velocity profile was 126.27 cm/s. To be viable for comparison with the other methods, a velocity limit, v_{max}, was set arbitrarily to 120 cm/s. The process was then repeated with implementation of v_{max} and the results are shown in Figure 6.

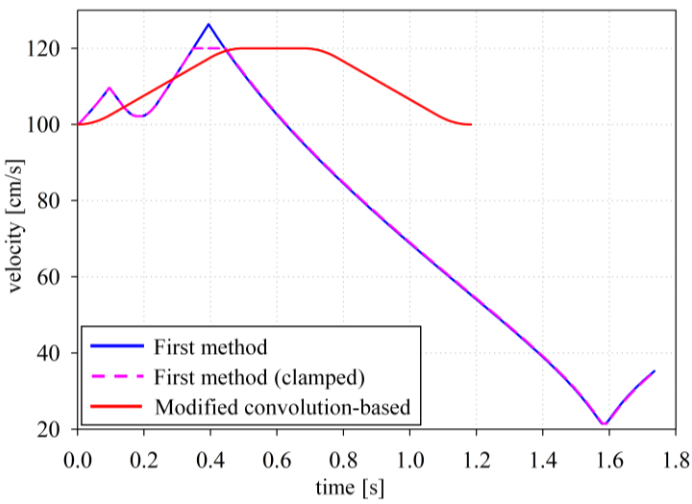

Figure 6. Comparison of Central Velocity Profiles in Time Domain

The maximum allowable velocity from the first method was clamped at 120 *cm/s* on the points where it exceeded the implemented velocity limit. From the figure, the initial and terminal velocities were clearly not affected after clamping however; the total travel time was increased by 0.02 *s* at 1.76 *s*.

The same curve was simulated using a different approach—modification of the the central velocity profile generated using a convolution operator [1]. Considering (2-b), the maximum acceleration value, $v^{(1)}_{max}$, was set equal to the maximum tangential acceleration from the previous method.

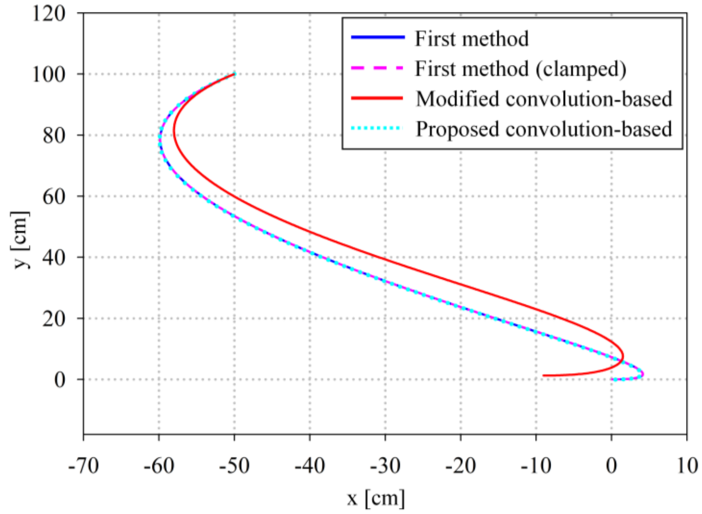

Figure 7. Generated Trajectories using Different Central Velocity Profiles

Since the planned path is based on a third-degree Bezier curve, the third position derivative, jerk, is needed and the value was determined to satisfy convolution conditions with a value of $v^{(2)}_{max}$=500 cm/s^3. The modified convolution-based central velocity was generated as shown in Figure 6. The result shows that the actual terminal velocity satisfies the required value.

To generate the trajectories found in Figure 7 above, the heading angles in radians at each section of the curved path is computed by:

$$d\theta = \tan^{-1}\frac{dy(u)}{dx(u)} \tag{8}$$

From (8), the velocity in each position differences in the x and y direction with respect to the given overall velocity is computed by robot kinematics [12] and is expressed as:

$$\begin{aligned} v_x(u) &= v(u)\cos(d\theta(u)) \\ v_y(u) &= v(u)\sin(d\theta(u)) \end{aligned} \tag{9}$$

Here, $v(u)$, represents the velocity profile to be used in driving the robot from the initial points. The trajectory $(x(u),y(u))$ is generated by adding the cumulative sum of the differences between all segments in both the x and y axes to the initial position values, x_i and y_i:

$$\begin{aligned} x(u) &= x_i + \sum_{u=0}^{n} v_x(u)dt \\ y(u) &= y_i + \sum_{u=0}^{n} v_y(u)dt \end{aligned} \tag{10}$$

where n is the total number of sample points and dt is the time that the robot would take to move from one point to another along the curve.

The results displayed by Figure 7, show that the trajectory generated from the first method, clamped or unclamped, could follow the designed high-curvature path while the modified convolution-based velocity profile produces a trajectory that shows the opposite result. Since the modified convolution-based velocity was generated in uniform sample time, the time differential of the trajectory generation methods was checked using the equation:

$$dt = \frac{ds(u)}{v(u)} \tag{11}$$

Figure 8 shows that the time differential for the robot movement along the path was found to be non-periodic for the first method. The modified convolution-based velocity method could also follow the high-curvature path if the time differential for each point is non-uniform. However, in a typical control system, periodic velocity commands is needed to make sure that the reference would be accurately tracked by the mobile robot. Also, real time operations require equal time differences between each sampling points in order for the system to be schedulable, for example cyclic tasks in real time systems.

In case of the first method, a large discrepancy between the actual and required terminal velocities were also shown from previous simulation results and according to [4], the solution for this problem is by shifting the Bezier polygon side d_2 to the left thus, the original planned path would be completely transformed. Therefore, this method would have difficulties to be implemented in practical situation; where uniform sampling time is needed for accurate robot control and schedulability, also in continuous path planning where the exact initial and terminal values are needed to generate consecutive trajectories.

On the other hand, the modified velocity using a convolution operator shows high accuracy in meeting the required terminal velocities despite of the error in reaching the target terminal point. Thus, linear interpolation to the modified convolution-based velocity values with respect to uniform sampling time is proposed to solve this problem.

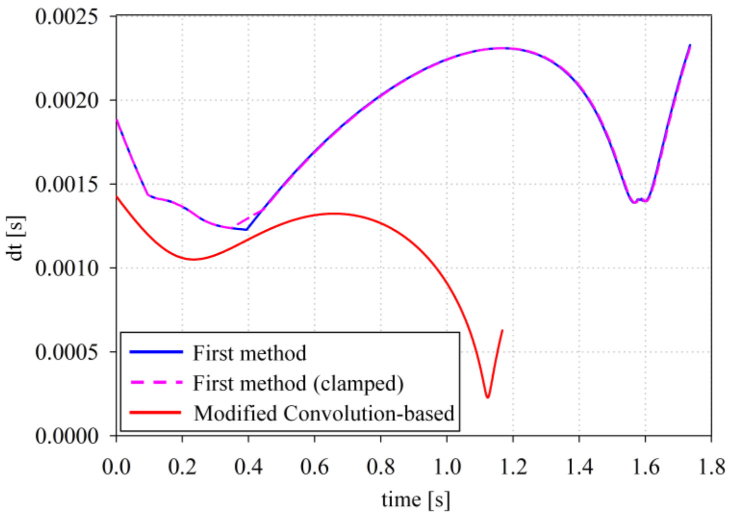

Figure 8. Time Difference between Each Points for Each Velocity Profile

The non-uniform time differential as shown in Figure 8 was taken by substituting the modified convolution-based central velocity, v_{conv}, to (11). The proposed velocity profile, v_{uni}, is generated by applying the following linear interpolation variation:

$$v_{uni}(t) = v_{conv}(t_{n-1}) + \frac{v_{conv}(t_n) - v_{conv}(t_{n-1})}{t_n - t_{n-1}}(t - t_{n-1}) \tag{12}$$

where the total time from integrating the result from (11) is denoted by t_n and the uniformly spaced t is defined as $t_{n-1} \leq t \leq t_n$.

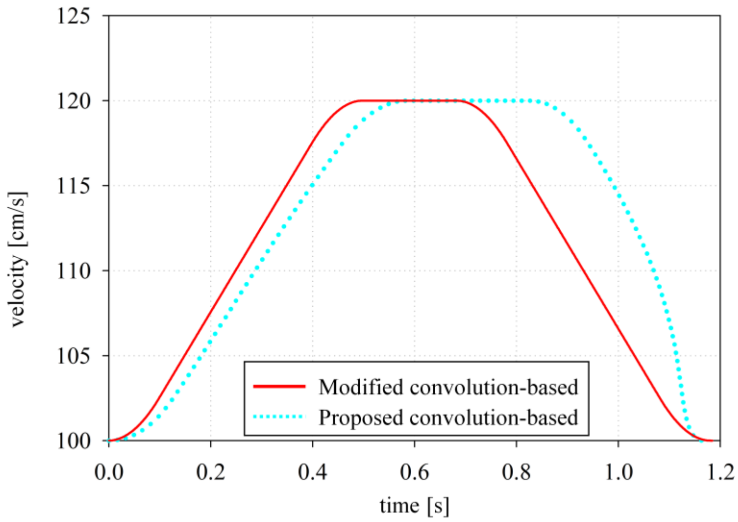

Figure 9. Convolution-based Velocity Profiles

To summarize the steps in generating the results in Figure 9:

- Calculate the total distance of the planned path, B_d, by integrating (5).
- Using the convolution operator in [10], and application of the modification presented in (7), the modified convolution-based velocity profile, v_{conv}, is generated.
- Non-uniform time differential of the convolution-based velocity is calculated using (11).
- The proposed convolution-based velocity profile, v_{uni}, is generated using an arbitrary uniform sample time using the variation of linear interpolation in (12).
- Direction angles and the velocity between each sample point in both x and y axes are computed by applying (8) and (9) to generate the numerically driven trajectory using (10).

After the suggested method has been applied the velocity profile after interpolation is shown in Figure 9 and the corresponding trajectory is shown in Figure 7. The total travel time calculated was reduced to 1.16 s. Thus, the new method was able to travel the planned path in the shortest time compared with the other approaches, while also demonstrating periodicity and being within the physical constraints of the mobile robot at the same time.

5. Conclusion

In this work, we proposed a path planning method that allows a mobile robot to travel along a cubic Bezier curve with high curvature while considering its physical limits.

We simulated a predetermined path based on a Bezier curve with high turning point curvature and its corresponding trajectory generation method using acceleration limits as suggested in [4]. The results displayed that although the velocity profile produced by this approach could make the mobile robot follow the high curvature path, this method exhibited relatively the longest travelling time, non-uniform time differential at each sampling point intervals, and huge disparity between the actual and required terminal velocities.

The same path was simulated using a modified convolution-based trajectory generation technique proposed in [1]. This approach was able to produce a velocity profile that met the required terminal values, however, it demonstrated incapability to follow the predetermined Bezier path.

For the general problem of generating a trajectory for the mobile robot to travel a path with high curvature, an approximation method using a variation of linear interpolation was presented and applied to the convolution-based velocity generation method to produce a velocity profile that compromises all the drawbacks of the previous methods.

After implementation of the proposed method, the generated trajectory was able to travel along the high curvature Bezier curve without violating any physical limits in the shortest time while also demonstrating periodicity. However, these results also discloses that further studies regarding operation of this method for two wheeled mobile robots while considering each of its actuator's physical limits is needed.

Furthermore, the proposed trajectory generation in this paper could be a ground for further development of path planning methods for real time obstacle avoidance.

Acknowledgements

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