Analysis of the reflected image by the cylindrical concave surface of the mobile display

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Abstract — Reflection from the display surface affects the user's viewing experience. Cylindrical concave displays had been reported to be used for TV and mobile smart phone. The position and magnification of reflection at the curved mobile display were determined by the user's position and the focal distance of the cylindrical concave surface of the curved display. If the focal position of the curved surface was smaller or nearer to the user position, the enlarged reflection of the user would block the lights from the surrounding, and the curved display would be effective to reduce the inconvenience due to reflection. As the curvature of the cylindrical surface was different for the vertical and horizontal directions, the reflected image was not located at one distance. Hence, the refractive power of the eye of the user would be difficult to adjust to the position of the reflected image, and the reflected image would look defocused or hazed even for the display surface of the specular reflection.

Keywords *— reflection, cylindrical concave, defocusing, haze.*

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1 Introduction

Recently, displays of the non-flat or flexible surfaces began to be used for the applications such as mobile displays, E-paper, and TV.[1–7] And there were many discussions going on about the merits of the curved display and why the curved surface was needed in the display application.

One possible merit was reported to be related to the reflection from the curved display screen.[8] The reflection of the ambient environment, in particular of ambient light sources, from the display screen creates the "disturbing information" that overlays the "useful information" from the display, thus reducing contrast and color and making it more difficult to read.[3,9,10] Recently, the measurement of the reflection and the reduction of the contrast due to the ambient light had been also reported for the curved TV.[11,12] In case of the flat panel display (FPD), the surface of the display had been treated to reduce the reflection ratio or have the characteristics of anti-glare. From the empirical observation, a concave phone display of a given curvature was reported to have the potential benefit that for certain viewing distances the user sees a magnified reflection of his or her face that may fill the entire screen area and thus entirely blocks the disturbing reflection of the ambient environment, although no mathematical analysis was provided.[8]

The purpose of the paper is to define the relationship between the display curvature and the viewing distance and specify the conditions where the disturbing reflections of the ambient environment are blocked from view. For this, the reflected image at the cylindrical concave surface of the mobile application was investigated, compared with that of the flat surface. The dependences of the position and the magnification of the reflected image were investigated with regard to the focal distance of the cylindrical surface and the position of the reflected object. As the curvature of the cylindrical surface was dependent with respect to the angle from the cylindrical axis, the effect due to the cylindrical shape was investigated in detail.

2 Methods

2.1 Reflection at the spherical surface

In this paper, only specular reflection was considered, and it was assumed that the surface of the reflection can be characterized by the focal distance f or the radius of the curvature. When an object of the size m was located at the distance s from the spherical mirror, the position s' and size m' of the reflected object was known to be dependent on the relative position between the position s and focal point F of the spherical mirror in the geometric optics.[13,14] Figure 1 illustrates the position and the size of the reflection in the spherical mirror. When s was smaller than focal distance f, the enlarged and upright reflection was located on the opposite side of viewer at the distance s' as illustrated in Fig. 1(a). At $s = f$, no reflection was formed, and at $s > 2f$, the reduced reflection was formed. Hence, the reflection of the object by the spherical concave surface can be enlarged or reduced, depending on the size of s/f.

2.2 Experimental setup for reflection at the curved surface

As the commercial smart phone has the shape of the cylindrical concave surface, the reflection by the cylindrical concave

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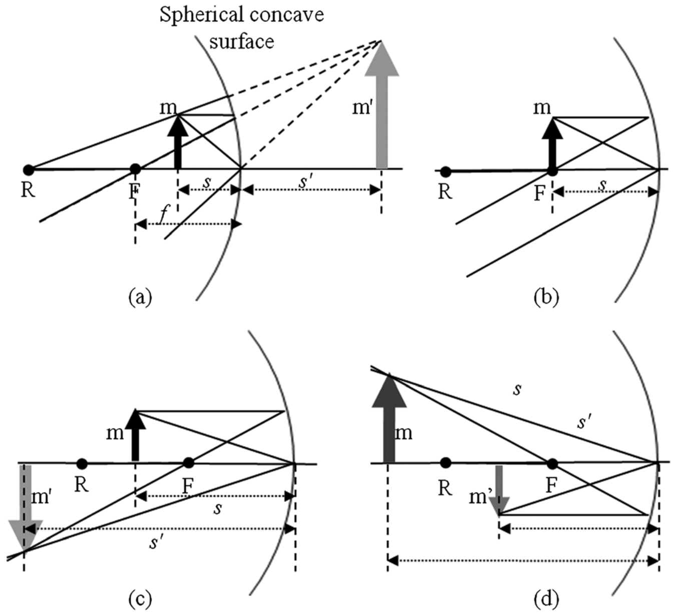

FIGURE 1 — Reflection at the spherical concave surface at (a) $s < f$, (b) $s = f$, (c) $s > f$, and (d) $s > 2f$. Distance of the objects from the surface and the focal distance are represented by s and f, respectively.

surface was investigated.[7] In the cylindrical concave surface, a direction exists where the curvature is infinite. This direction was defined as the cylindrical axis. Figure 2 illustrates the schematic setup to photograph the reflected image. The cylindrical axis was selected as the vertical direction. Hence, the reflection along the vertical direction would be the same as the reflection at the flat mirror, while the magnification and inversion of the reflected image by the curved surface will be noticeable along the horizontal direction. In order to observe the magnification and inversion of the reflected image by the curved sample display, an image file of the mesh pattern was used. The mesh pattern consisted of the squares of equal sizes for the observation of the magnification of the reflected image. Inside the mesh pattern, dark solid triangles

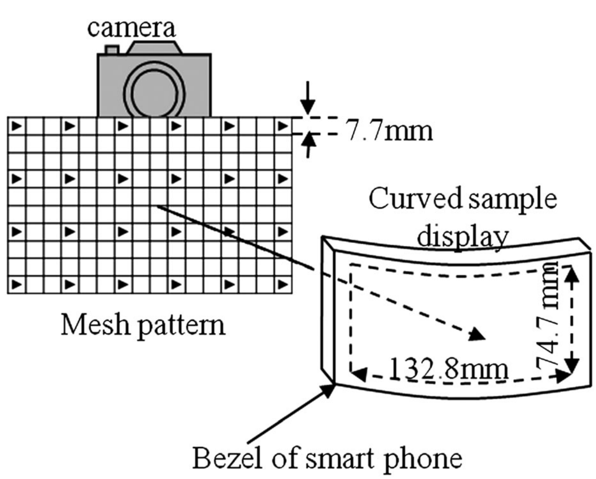

FIGURE 2 — Schematic setup to photograph the reflection by the sample display of the cylindrical concave surface. The axis of the cylindrical surface was along the vertical direction. A mesh pattern was placed in front of the curved display, and its reflection was photographed by a camera.

of "▸" were located for the observation of the inversion of the reflected image along the horizontal direction. The image file of this mesh pattern was used as the input data for an FPD of the diagonal size of 9.7 in. On the FPD of the diagonal size of 9.7 in. size, each square of the mesh pattern was represented as the size of 7.7 mm. This FPD was placed in front of the curved sample display where the center positions of this FPD and the curved display were approximately aligned on the same line. The reflection of the mesh pattern by the curved sample display was photographed by a camera. A camera was located at the upper side of the FPD and slightly inclined from the horizontal direction. Considering that the eyes and the face of the user were at the same viewing distance, the camera and this FPD for the mesh patterns were selected to be located at the same distance from the curved sample display.

2.3 Theory of the magnification of the reflection at the cylindrical concave surface

When a user uses the sample mobile display, the reflection of the user's face will appear on the screen. In case of the flat surface, the absolute values of the position s_f' and the size m_f' of the reflection are the same as the absolute values of the position s and the size m of the object as illustrated in Fig. 3(a). Hence, angle A that the reflection of the user's face occupies at the flat surface can be represented as Eq. ((1))

$$\tan A = \frac{m_f'}{s + \left|s_f'\right|} = \frac{m}{2s} \tag{1}$$

Here, m corresponds to the horizontal size of the user's face. And angle C is defined as the angle that the horizontal length of the display occupies in the view of the user.

In case of the cylindrical concave surface where the cylindrical axis of the sample display is vertical, the enlargement or the reduction of the reflected image occurs only along the horizontal direction. From the spherical mirror formula, the position of the object s and the reflection s' is represented by the following equation.[13,14]

$$1/s + 1/s' = 1/f \tag{2a}$$

And s' and the magnification m'/m can be represented by the following equations.

$$s' = \frac{s}{(s/f - 1)} \tag{2b}$$

$$\frac{m'}{m} = \frac{s'}{s} = \frac{1}{(s/f - 1)} \tag{2c}$$

Hence, angle B that the reflected face occupies on the curved surface can be written as Eq. (2d).

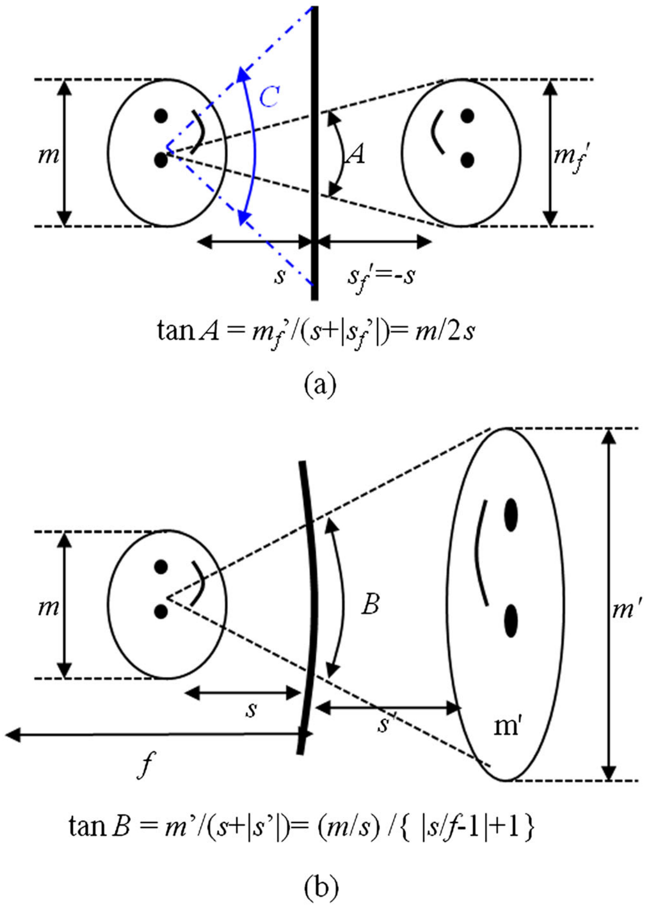

FIGURE 3 — Angles that the reflected image of the user's face occupies at (a) the flat surface and (b) the curved surface at $s < f$. User is located at the distance of *s*. Angle *C* is defined as the angle that the display horizontally occupies in the view of the user.

$$\tan B = \frac{m'}{s+|s'|} = m\frac{s'}{s} \times \frac{1}{s+|s'|} = \frac{m}{2s}\frac{2}{(|s/f-1|+1)}$$
$$= \frac{2\tan A}{(|s/f-1|+1)} \qquad (2d)$$

When $s/f < 1$, Eq. ((2d)) can be written as $\tan B = 2f/s \times \tan A > \tan A$. It means that the angle occupied by the reflected user's face is always larger at the curved concave surface than at the flat surface if the distance s is smaller than f as illustrated in Fig. 3(b). In that case, the reflection of the horizontally enlarged user's face is expected to be effective to block the reflection of the various light sources on the surrounding.[8]

Figure 4 shows the relation between s/f and s'/f for the flat and concave surfaces. Distances of the reflection by the flat surface and the curved surface were represented as s_f' and s_c', respectively. When s was smaller than f, the reflected image enlarged along the horizontal direction was formed at the opposite site of the viewer. (−) sign was used to represent the reflection at the opposite side of the viewer. As long as s was smaller than $2f$ in the curved display, the magnification $|s'/s|$ was always larger than 1, and the reflection by the curved

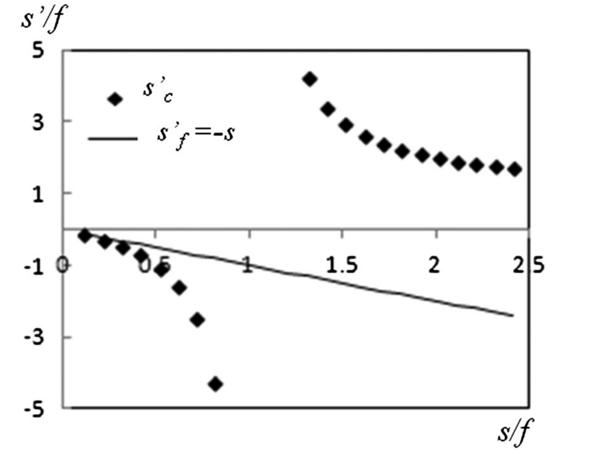

FIGURE 4 — The distances of the reflected images by the flat surface and the curved surface. Distances of the reflection by the flat surface and the curved surface were represented as s_f' and s_c', respectively. Horizontal and vertical axes represent the distance to the user position and the reflected image divided by the focal distance *f*, respectively.

surface would be located farther from the position of the display compared with the reflection by the flat surface.

2.4 Theory of the astigmatic effect of the reflection at the cylindrical concave surface

The position of the reflected image for the spherical surface can be determined by the focal distance f. In the cylindrical surface that the axis of the cylinder is along the vertical direction, the sample is curved along the horizontal direction by the focal distance f as illustrated in Fig. 5(a). Hence, the focal distance of the cylindrical surface can be represented as the function of the angle ϕ from the cylindrical axis where $f_c(\phi) = f/\sin^2(\phi)$.[15] Then, the reflected image by the cylindrical surface is formed at distance s' of Eq. ((2b)) at the focal distance of $f_c(\phi)$ at the angle ϕ. In case of the vertical direction of $\phi = 0°$, $f_c(\phi)$ become infinite, and the reflected image is formed at the distance of $(-s)$ like the reflection at the flat surface. In case of the horizontal direction of $\phi = 90°$, $f_c(\phi)$ become f, and the reflected image is formed at the distance of $s/(s/f-1)$. Therefore, the position of the reflection at the cylindrical surface cannot be defined as one point but exists in the range of $(-s)$ and $s_c' = s/(s/f-1)$ as illustrated in Fig. 5.

The phenomena that the reflected image is not at one position due to the cylindrical concave mirror are similar to those of astigmatism by the toric lens.[14–17] Toric lens was represented as the combination of the spherical lens and the cylindrical lens and was characterized by the two refractive power and two orientations perpendicular to each other. When the parallel rays pass through the toric lens, the rays form two mutually perpendicular focal lines, instead of one focal point. As two refractive powers of the toric lens determined the positions of these two focal lines, the amount

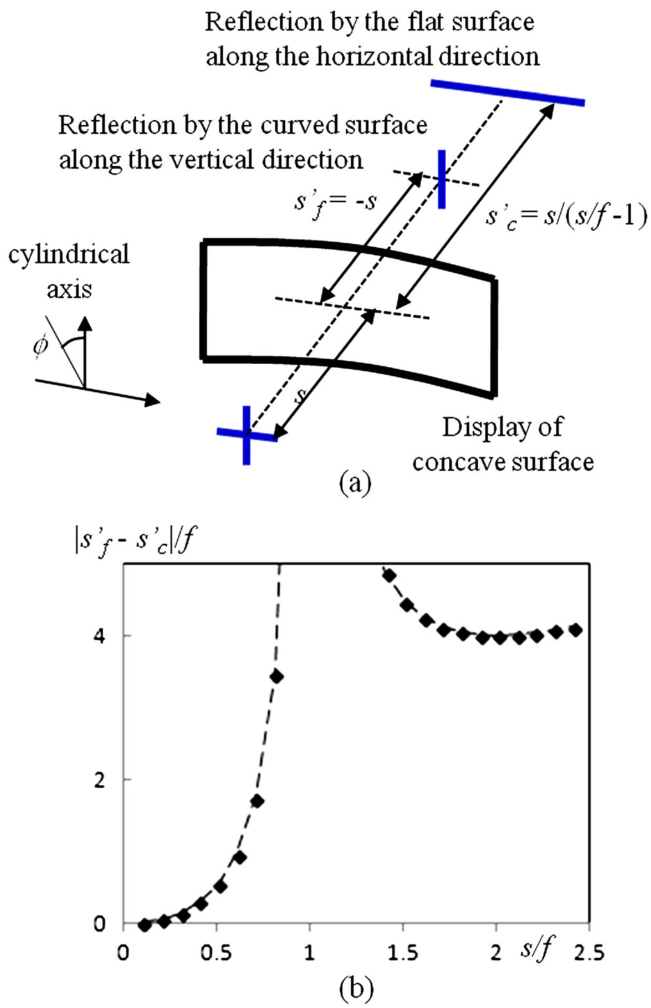

FIGURE 5 — (a) The relation between the position *s* of the object and the position *s*′ of the reflection at the cylindrical concave surface. Axis of the cylinder was parallel to the vertical direction. (b) The difference between the distances of the reflection by the flat surface and the curved surface of the focal distance *f*. Distances of the reflection by the flat surface and the curved surface were represented as s_f' and s_c', respectively. Horizontal axes represent the distance to the user position divided by the focal distance *f*.

of astigmatism was represented as the distance between these focal lines. If the amount of astigmatism was large, users were known to be difficult to observe the sharp focused image as the eye could not focus on the one position.

Figure 5(b) illustrates the difference between the normalized distances of the reflection by the flat surface and the curved surface of the focal distance *f*. As the range of $f_c(\phi) = f/\sin^2(\phi)$ was from *f* to infinite in the cylindrical concave surface, this difference determined the range of position of the reflected image. This difference remain larger than 4 as *s/f* increased. If this difference due to the cylindrical shape is too large, the user is expected to have difficultly observing the sharp focused reflected image even if the eye tried to focus on the reflected image by the change of the refractive power of the eye.

3 Results

Figure 6 shows the photos of the reflection of the mesh pattern by the curved sample display, taken in the setup of Fig. 2. The specifications of the sample of the curved display were a diagonal size of 6 in. (152.5 mm), a resolution of 1280 pixels by 720 pixels, an active area of 132.8 mm by 74.7 mm, and curvature of 700 mm.[7] The shape of the sample display was concave cylinder. The sample display was curved only along the

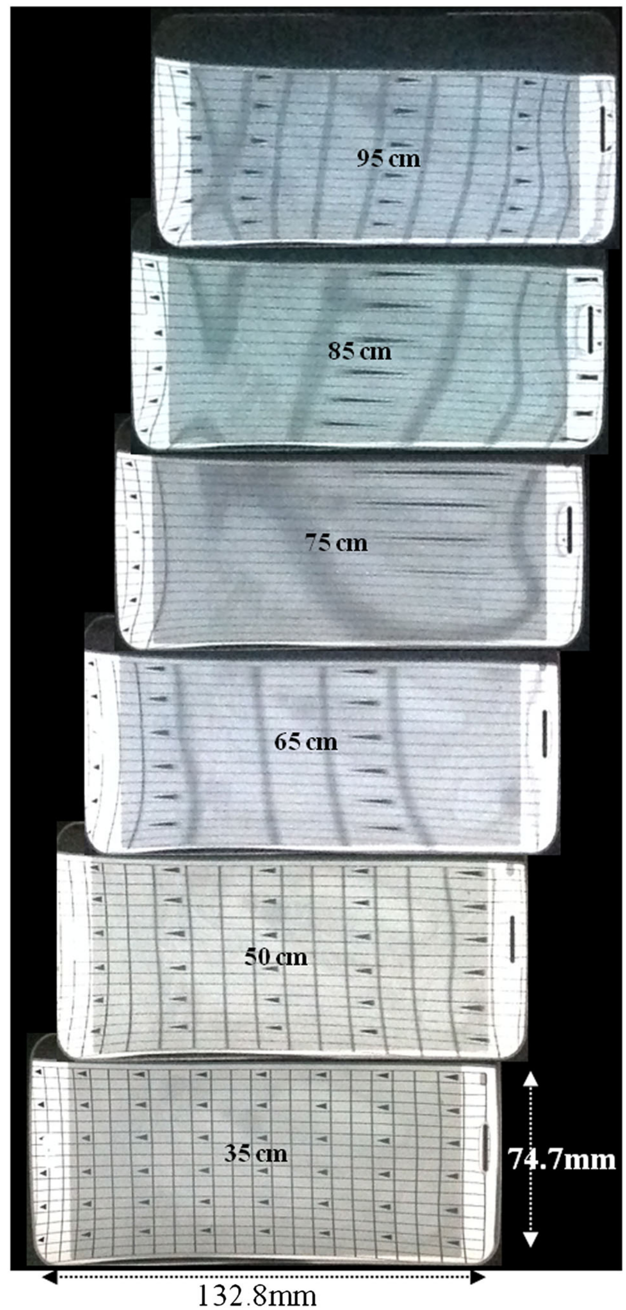

FIGURE 6 — Photos of the reflection of the mesh pattern image by the curved sample display at the various distances of *s*. The conditions of the distances were noted at the center of each photo. Active area of the curved display corresponded to the region of the slightly darker rectangle inside the bezel of the sample smart phone.

horizontal direction of 132.8-mm length and was not curved along the vertical direction of 74.7-mm height. Distances at the center of each photo represented the viewing distance s from the mesh pattern to the curved sample display. A camera was located at the upper side of the mesh pattern and at the same distance from the curved sample display as the mesh pattern. Viewing distances were changed from 35 to 95 cm by the step of 10 cm. At s = 35 cm, the reflection of the mesh pattern was slightly enlarged along the horizontal direction, and the squares of the mesh pattern became rectangular in the photo. As the viewing distance s became larger than 35 cm, the horizontal magnification of the reflection also increased until s reached 75 cm. Around the distance of 75 cm, the reflected image was barely formed. When the distance increased further from 75 cm, the horizontal magnification started to decrease. The direction of dark solid triangle changed from "◂" to "▸", as the distance s changed from 35 to 95 cm. So the inversion of the reflected image occurred when s was larger than 85 cm. Photos of the reflection in Fig. 6 show the trends of the magnification, which were expected with the focal distance of around 75 cm for the cylindrical concave surface with the vertical cylindrical axis.

As the mesh pattern was too simple to observe the defocusing due to the focal positions of the reflected image, the photos of the real object were also taken. Figure 7 show the photos of the reflection of a camera by the curved sample display at the distance s of 20 and 40 cm. The camera was set at the auto-focus and placed toward the center of the curved sample display, which was not operating. Photos of Fig. 7 show the enlarged reflection of the camera along the horizontal direction as the reflection of the circular camera lens was observed to be elliptical. In the photo of Fig. 7(b) where the camera was located at the distance of s = 40 cm, the reflection of the camera was slightly out of focus, while the bezel of the smart phone was in focus.

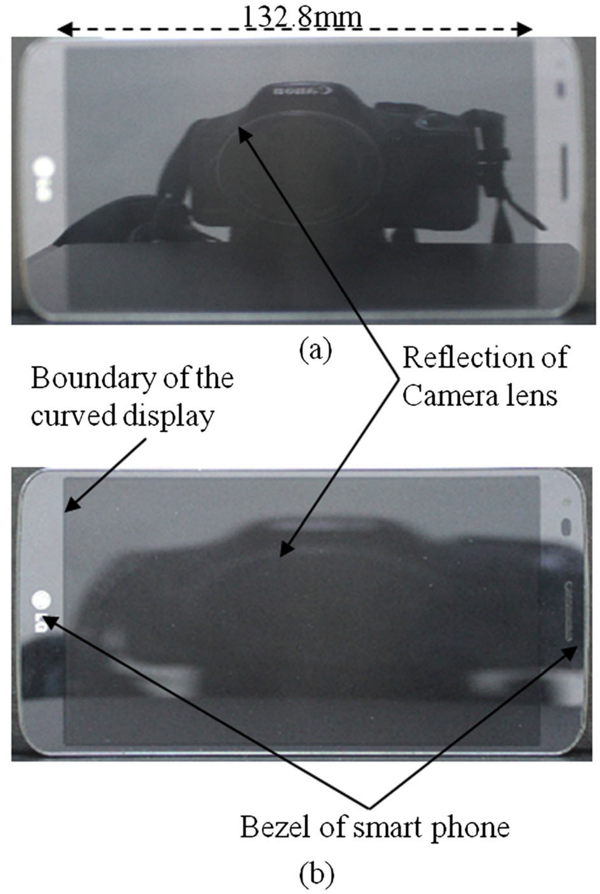

FIGURE 7 — Photos of the reflection of a camera by the curved sample display. Camera was set at the auto-focus. Distance s from the end of the camera lens to the curved sample display was (a) 20 cm and (b) 40 cm. Active area of the curved display corresponded to the region of the slightly darker rectangle inside the bezel of the sample smart phone.

4 Discussion

The reflection from the display screen creates the "disturbing information" that overlays the "useful information" from the display. The "useful information" from the display was located at the positions of the display, that is, at the same distance as the bezel of the display. The photo of Fig. 7(b) means that the user would observe the defocused reflection as the user's eyes focused on the "useful information" from the display. The observation of the defocused reflection can be attributed to depth of field (DOF) and the astigmatic effect.

Depth of field was defined as the distance range that the eye or the camera could clearly observe objects simultaneously and located around the positions of the objects that the user's eye stare at. When the refractive power of the user's eye changes to clearly observe the image data on the curved display, DOF is located around the curved display. If the distance between the display and the reflection by the surface is larger than DOF, the reflection will look defocused to the eye as the reflected image is located outside DOF. As an example, the positions of the reflected at the viewing distance of 20, 30, and 40 cm were calculated in Table 1 for the focal distance of 75 cm. The distance s_c' between the display and the reflection by the curved surface was larger than the distance $-s$ between the display and the reflection by the flat surface. So it became more probable that the reflection by the curved surface was located outside DOF.

If the reflected image was located at one specific position, the user could observe the clear reflected image by staring at

TABLE 1 — Calculated angles A and B that the reflected face occupied for the flat and curved surfaces for the various viewing distances.

Viewing distance (cm)	s_c' (cm)	Angle A (°)	Angle B (°)	Angle C (°)
20	27	21	24	34
30	50	14	18	24
40	86	11	15	18

Angles A, B, and C were defined in Fig. 3 and Eqs (1) and (2a), (2b), (2c), (2d). The focal distance and the horizontal size of the display were selected to be 75 and 13.3 cm, which were the same as the sample display. The face size was selected to be 15 cm.

the reflected image. In case of the reflection at the cylindrical reflection where the astigmatic effect was not negligible, the user would have difficulty observing the clear reflected image even if the user tried to stare at the reflected image.

This phenomenon may be compared with those of the reflection as the hazed surface. Figure 8(a) illustrates the rays coming from the reflected image where the position of the reflection by cylindrical surface existed in the range of $(-s)$ and $s_c' = s/(s/f - 1)$. As the reflected image was not at one position, the camera or the eye of the user would see the rays where the directions of the rays from the reflected image were distributed along the direction toward the user or the camera. Haze surface was defined as the surface that light scattered around specific direction as illustrated in Fig. 8(b).[10] Even if there was only a specular reflection on the display surface, the display of the cylindrical shape may cause the reflection similar to the haze surface such that the ray directions are around the specific direction.

The effect of the reflection was determined by the user position and the focal distance. When the distance s from the user's position to the display was smaller or near to the focal distance f of the curved concave display, the magnification became larger than one. As an example, the angles that the reflected face occupied at the viewing distance of 20, 30, and 40 cm were calculated in Table 1. Angles A and B represent the calculated angles that the reflected face occupied at the flat surface and the curved surface of the focal distance of 75 cm for the face size of 15 cm. Angle C represents the angle that the display of the horizontal length of 132.8 mm occupies in the view of the user. In Table 1, for the larger viewing distance, angle A for the curved surface became larger than angle B for the flat surface. The result shows that the reflection of the enlarged user's face would be effective than the flat surface to block the disturbing reflection of the ambient environment.[8] As the ratio of $|s'/s|$ became larger than one, it also became more probable that the user could not observe the sharp reflection due to the limited size of DOF or the astigmatic effect of the cylindrical surface.

If the focal distance of the curved concave display is selected to be quite a bit smaller than the distance to the user s, the inverted reduced reflection of the user's face by the curved display will be located near to the display as illustrated in Fig. 1(d). In that case, the reflection of the reduced user's face would not be effective in blocking the lights from the ambient environment compared with the reflection by the flat display. Yet, the various light sources on the surrounding would still look less sharp owing to the reflection at the cylindrical shape.

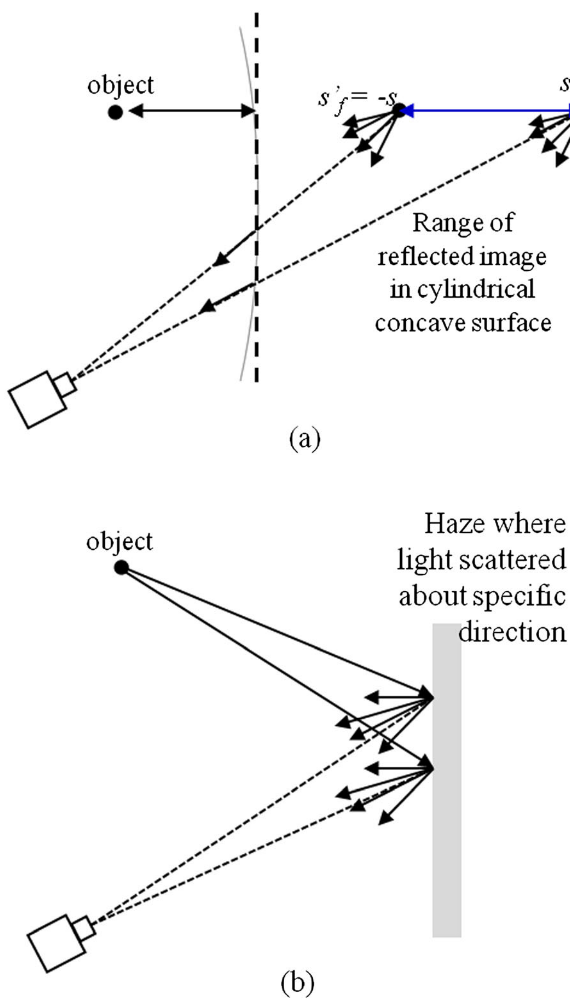

FIGURE 8 — (a) The rays coming from the reflected image where the position of the reflection by cylindrical surface exists in the range of $(-s)$ and s_c'. (b) Haze phenomenon that that light scattered around specific direction.

5 Conclusion

The reflection on the mobile display of the cylindrical concave surface was investigated with regard to the user position and the focal distance of the curved display. The position of the reflected image was determined by the distance to the sample and the focal distance of the curved surface. If the distance of the user's position was selected to be nearer to the focal distance f, the reflection of the enlarged user's face would be effective to block the reflection of the various light sources on the surrounding.[8] If the focal distance f was selected to be much smaller than the distance to the user's position, the magnification of the reflection would be less than one, so the blocking by the user' face would not be effective.

As the focal distance of the cylindrical surface depends on the angle ϕ from the cylindrical axis, the position of the reflected image cannot be determined as one position. Therefore, the user or the camera would have difficulty observing the focused sharp reflection similar to the reflection at the hazed surface.

The viewing distance and the focal distance of the curved surface determine the magnification and the sharpness of the reflected image, which in turn affects the viewing experience of the user. Hence, the magnification and the sharpness of the reflected image should be considered in deciding the appropriate size of the viewing distance and the focal distance of the curved surface.

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