# Kinematics Modelling for Omnidirectional Rolling Robot

**Soo-Yeong Yi**

**Abstract** A ball-shaped mobile robot, called a ballbot, has a single point of contact with the ground. Thus, it has low energy consumption for motion because of the reduced friction. This paper presents the systematic kinematics modelling for a type of ballbot with omnidirectional motion capability. This kinematics modelling describes the velocity relationship between the driving motors and the robot body for the motion control of the robot.

## 1 Introduction

A ball-shaped robot has a single point of contact with the ground, which reduces its friction with the ground. A ball-shaped robot is generally called a ballbot. In comparison with a conventional wheeled mobile robot, a ballbot consumes less energy for motion because of the reduced friction [1]. There are two types of ballbots, as illustrated in Fig. 1. The ballbot shown in Fig. 1a has a cylindrical body on the top of a ball [2, 3]. This cylindrical body has driving motor and wheel assemblies in contact with the exterior of the ball to exert a driving force. In contrast, the ballbot shown in Fig. 1b has a pure spherical shape and contains a driving mechanism inside the ball [4]. The driving mechanisms can be classified into two types: (i) the wheeled platform type (Fig. 1b) [4] and (ii) the pendulum type (Fig. 2) [1].

The ballbot shown in Fig. 1a has many motion control difficulties because its posture is essentially unstable. In contrast, the pure ball-shaped robots shown in Fig. 1b and Fig. 2 are inherently stable, so the motion control is relatively stable.

S.-Y. Yi (✉)
Department of Electrical and Information Engineering, Seoul National University of Science and Technology, Seoul, Republic of Korea
e-mail: suylee@seoultech.ac.kr

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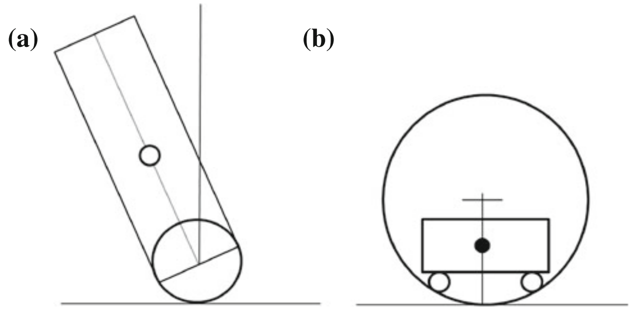


Fig. 1 Types of the ballbot. **a** Cylindrical robot body. **b** Spherical robot body

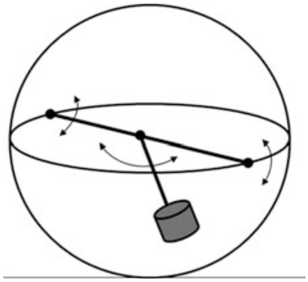

**Fig. 2** Pendulum type driving mechanism

However, these ballbots still have a motion control problem because the driving mechanism cannot provide omnidirectional motion capability.

In this paper, the systematic kinematics modelling for a ballbot with a wheel-type driving mechanism inside a ball is addressed for the motion control of the ballbot. More specifically, the driving mechanism is a platform with three Swedish wheels, so the ballbot has omnidirectional motion capability without nonholonomic constraints. Thus, the motion control of the ballbot becomes comparatively simple.

## 2 Differential Motion Between Coordinate Frames

The kinematics modelling can be described by the velocity relationship between the active driving motor and the robot body. When the transformation between two coordinate frames, **B** and **C**, is given as $T_B^C$, the relationship of the differential motions between the coordinate frames shown in Fig. 3 is described as

$$\Delta^C = T_B^{C-1} \cdot \Delta^B \cdot T_B^C, \tag{1}$$

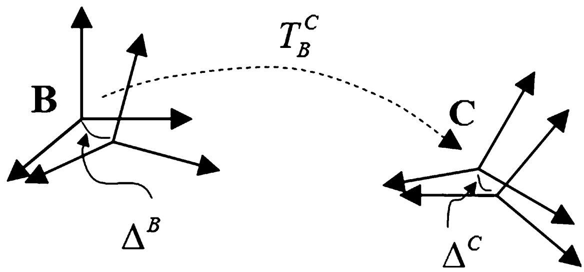


**Fig. 3** Relationship of differential motions between coordinates frames

where $\Delta^B$ and $\Delta^C$ denote the differential motions in the corresponding coordinate frames [5]. The differential motion ($\Delta$) implies a velocity transform if the motion occurs in a small time interval, $\delta t$, and can be written as

$$\Delta = \begin{bmatrix} 0 & -\delta_z & \delta_y & d_x \\ \delta_z & 0 & -\delta_x & d_y \\ -\delta_y & \delta_x & 0 & d_z \\ 0 & 0 & 0 & 0 \end{bmatrix}, \tag{2}$$

where $\bar{\boldsymbol{\delta}} = [\,\delta_x \quad \delta_y \quad \delta_z\,]^t$ and $\bar{\mathbf{d}} = [\,d_x \quad d_y \quad d_z\,]^t$ denote the rotational and translational differential motions, respectively.

The transformation $T_B^C$ is represented by column vectors as (3).

$$T_B^C = [\bar{\mathbf{n}} \quad \bar{\mathbf{o}} \quad \bar{\mathbf{a}} \quad \bar{\mathbf{p}}]. \tag{3}$$

Then, from (1) and (2), each component of the differential motions in coordinate system **C** becomes (4-1) and (4-1).

$$\bar{\boldsymbol{\delta}}^C = \begin{bmatrix} \delta_x^C & \delta_y^C & \delta_z^C \end{bmatrix}^t = \begin{bmatrix} \overline{\boldsymbol{\delta}^B} \cdot \bar{\mathbf{n}} & \overline{\boldsymbol{\delta}^B} \cdot \bar{\mathbf{o}} & \overline{\boldsymbol{\delta}^B} \cdot \bar{\mathbf{a}} \end{bmatrix}^t, \tag{4 – 1}$$

$$\begin{aligned} \bar{\mathbf{d}}^C &= [\,d_x^C \quad d_y^C \quad d_z^C\,]^t \\ &= \left[\bar{\mathbf{n}} \cdot \left(\overline{\boldsymbol{\delta}^B} \times \bar{\mathbf{p}} + \overline{\mathbf{d}^B}\right) \quad \bar{\mathbf{o}} \cdot \left(\overline{\boldsymbol{\delta}^B} \times \bar{\mathbf{p}} + \overline{\mathbf{d}^B}\right) \quad \bar{\mathbf{a}} \cdot \left(\overline{\boldsymbol{\delta}^B} \times \bar{\mathbf{p}} + \overline{\mathbf{d}^B}\right)\right]^t. \end{aligned} \tag{4 – 2}$$

In the above equations, "·" and "×" denote the inner product and outer product, respectively, of the vectors.

## 3 Structure of Ballbot and Assignment of Coordinate Frames

The structure of the ballbot considered in this paper is shown in Fig. 4a. The driving system inside the ball is an omnidirectional mobile platform having three Swedish wheels with 120° spacing. Each wheel is normal to the interior tangential

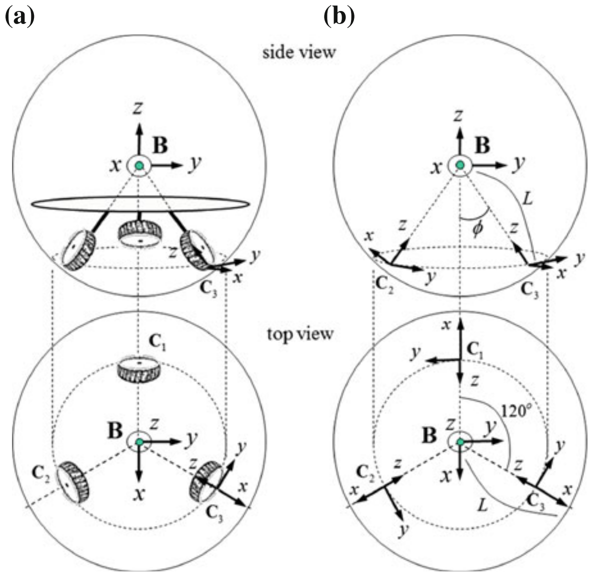


**Fig. 4** Structure and coordinate assignment of proposed ballbot. **a** Structure of ballbot. **b** Coordinate frames

plane of the ball at the point of contact. The driving force of the ball comes from the friction between the wheel and the interior surface of the ball. The coordinate frame assignment of the ballbot is depicted in Fig. 4b. In this figure, the coordinate frame at the centre of the ball is denoted as **B**, which is the inertial coordinate frame attached to the driving platform. The coordinate frames at the contact points of the wheels on the inside surface of the ball are represented as $\mathbf{C}_i$, $i = 1, 2, 3$.

To derive the kinematics model, it is assumed that the motion of the ballbot is quasi-static. This quasi-static motion implies that the motion has a constant velocity and, as a consequence, the driving platform inside the ballbot maintains level always when in motion. This assumption simplifies the motion of the ballbot by disregarding the dynamics effects and gives the velocity relationship between each driving wheel and the robot body. The motion of the ballbot can be described by the translational and rotational velocities at ground contact **G**. Here, the translational velocity implies the differential motion on the horizontal $x$–$y$ plane, and the rotational velocity denotes the differential motion about the vertical $z$ axis at the ground contact (Fig. 5). It should be noted that the translational and rotational velocities $(v_{xy}^G, \omega_z^G)$ of the ballbot are the same as the velocities of the inertial coordinate frame, **B**, of the platform $(v_{xy}^B, \omega_z^B)$. Thus, the motion kinematics of the ballbot can be represented by the velocity relationship between coordinate frame **B** and each wheel coordinate frame $\mathbf{C}_i$, $i = 1, 2, 3$.

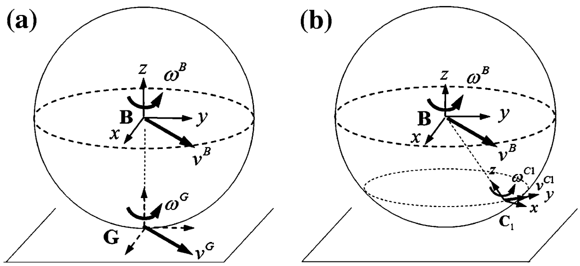


**Fig. 5** Motion of ballbot $(v_{xy},\ \omega_z)$: **a** motion at ground contact **G** and at centre of ball **B**; **b** motion at centre of ball **B** and at each driving wheel $\mathbf{C}_i$

## 4 Velocity Relationship Between Wheel and Robot Body

From Fig. 4b, coordinate transformations from **B** to $\mathbf{C}_i$, $i = 1,\ 2,\ 3$ are given as follows:

$$T_B^{C_1} = Rot(z,\pi)\cdot Rot(y,-\phi)\cdot Trans(z,-L) = \begin{bmatrix} -c\phi & 0 & -s\phi & -Ls\phi \\ 0 & -1 & 0 & 0 \\ s\phi & 0 & c\phi & -Lc\phi \\ 0 & 0 & 0 & 1 \end{bmatrix}, \qquad (5-1)$$

$$T_B^{C_2} = Rot\left(z,-\frac{\pi}{3}\right)\cdot Rot(y,-\phi)\cdot Trans(z,-L) = \begin{bmatrix} -\frac{1}{2}c\phi & \frac{\sqrt{3}}{2} & -\frac{1}{2}s\phi & \frac{1}{2}Ls\phi \\ -\frac{\sqrt{3}}{2}c\phi & \frac{1}{2} & \frac{\sqrt{3}}{2}s\phi & -\frac{\sqrt{3}}{2}Ls\phi \\ s\phi & 0 & c\phi & -Lc\phi \\ 0 & 0 & 0 & 1 \end{bmatrix}, \qquad (5-2)$$

$$T_B^{C_3} = Rot\left(z,\frac{\pi}{3}\right)\cdot Rot(y,-\phi)\cdot Trans(z,-L) = \begin{bmatrix} \frac{1}{2}c\phi & -\frac{\sqrt{3}}{2} & -\frac{1}{2}s\phi & \frac{1}{2}Ls\phi \\ \frac{\sqrt{3}}{2}c\phi & \frac{1}{2} & -\frac{\sqrt{3}}{2}s\phi & \frac{\sqrt{3}}{2}Ls\phi \\ s\phi & 0 & c\phi & -Lc\phi \\ 0 & 0 & 0 & 1 \end{bmatrix}. \qquad (5-3)$$

In (5-1)–(5-3), $L$ denotes the radius of the ball, $\phi$ is the zenith angle as shown in Fig. 4b, and $s\phi$ and $c\phi$ represent $\sin(\phi)$ and $\cos(\phi)$, respectively. The transformations in (5-1)–(5-3) can be written by column vectors as (3). In the case of $i = 1$ for example, the relationship between each component of the differential motion of (2) can be obtained as follows. It should be noted that $d_z^B = 0$ and $\delta_x^B = \delta_y^B = 0$ at first. From (1), (2), (3), and (5-1) through (5-3), the differential motion components are given as follows by equating both sides of (1):

$$\bar{\boldsymbol{\delta}}^{C_1} = \begin{bmatrix} \delta_x^{C_1} & \delta_y^{C_1} & \delta_z^{C_1} \end{bmatrix}^t = \begin{bmatrix} s\phi\,\delta_z^B & 0 & c\phi\,\delta_z^B \end{bmatrix}^t, \qquad (6-1)$$

$$\bar{\mathbf{d}}^{C_1} = \begin{bmatrix} d_x^{C_1} & d_y^{C_1} & d_z^{C_1} \end{bmatrix}^t = \begin{bmatrix} -c\phi\,d_x^B & L s\phi\,\delta_z^B - d_y^B & -s\phi\,d_x^B \end{bmatrix}^t. \qquad (6-2)$$

It should be noted that the active motion at each wheel is only $v_y^{C_i}$ according to the coordinate assignment in Fig. 4b, which can be generated by the driving motor of the wheel. From (6-2), $v_y^{C_1}$ is given as follows:

$$v_y^{C_1} = L s\phi\,\omega_z^B - v_y^B\,. \qquad (7)$$

Equation (7) represents the velocity relationship between a wheel and the robot body.

Similarly, from (1)–(6-2), the relationship between the velocity motion at **B** and the active motion of each wheel can be obtained as

$$v_y^{C_2} = Ls\phi\omega_z^B + \frac{\sqrt{3}}{2}v_x^B + \frac{1}{2}v_y^B\,, \qquad (8)$$

$$v_y^{C_3} = Ls\phi\omega_z^B - \frac{\sqrt{3}}{2}v_x^B + \frac{1}{2}v_y^B\,. \qquad (9)$$

Finally, from (7)–(9), the velocity kinematics of the ballbot in matrix form is described as

$$\begin{aligned} \begin{bmatrix} v_y^{C_1} \\ v_y^{C_2} \\ v_y^{C_2} \end{bmatrix} &= \begin{bmatrix} 0 & -1 & Ls\phi \\ \frac{\sqrt{3}}{2} & \frac{1}{2} & Ls\phi \\ -\frac{\sqrt{3}}{2} & \frac{1}{2} & Ls\phi \end{bmatrix} \begin{bmatrix} v_x^B \\ v_y^B \\ \omega_z^B \end{bmatrix} \rightarrow \\ \begin{bmatrix} v_x^B \\ v_y^B \\ \omega_z^B \end{bmatrix} &= \begin{bmatrix} 0 & -1 & Ls\phi \\ \frac{\sqrt{3}}{2} & \frac{1}{2} & Ls\phi \\ -\frac{\sqrt{3}}{2} & \frac{1}{2} & Ls\phi \end{bmatrix}^{-1} \begin{bmatrix} v_y^{C_1} \\ v_y^{C_2} \\ v_y^{C_3} \end{bmatrix} \end{aligned}. \qquad (10)$$

## 5 Conclusions

The ballbot considered in this paper uses an omnidirectional mobile platform with three Swedish wheels inside it as a driving mechanism and has several advantages over conventional ballbots: free motion without nonholonomic constraints, an inherently stable posture, and low energy consumption for motion. Kinematics modelling as an equation of motion is an essential prerequisite for motion control. Systematic kinematics modelling was addressed in this paper, which described the velocity relationship between the driving motors and the robot body for the motion control.

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